Carnegie Mellon University
21-122, Summer Session 2012

Second Homework, due July 16th

1. Use the Trapezoidal Rule to approximate the following integrals:

   I) $\displaystyle\int_1^2 \frac{\ln(x)dx}{1+x}$, $n = 10$

   II) $\displaystyle\int_1^2 e^{1/x}dx$, $n = 4$

   III) $\displaystyle\int_1^5 \frac{\cos(x)}{x}dx$, $n = 8$

2. Determine wether each integral is convergent or divergent. Evaluate those which are convergent:

   I) $\displaystyle\int_1^\infty \frac{1}{(3x+1)^2}dx$

   II) $\displaystyle\int_{-\infty}^{-1} \frac{1}{\sqrt{2-x}}dx$

   III) $\displaystyle\int_{-\infty}^{\infty} xe^{-x^2}dx$

   IV) $\displaystyle\int_0^\infty \frac{x\arctan(x)}{(1+x^2)^2}dx$

   V) $\displaystyle\int_0^3 \frac{1}{x\sqrt{x}}dx$

3. Determine wether the integral is convergent or divergent:

   I) $\displaystyle\int_1^\infty \frac{\cos^2(x)}{x^3}dx$

   II) $\displaystyle\int_1^\infty \frac{dx}{x+e^{2x}}dx$

4. Find the length of the curve $y = 1 + 6x^{3/2}$, when x varies from 0 to 1.

5. Consider the curve given by the equation $y^3 = x^2$

   a) Sketch the curve.

b) Set up two integrals for the arc length from $(0,0)$ to $(1,1)$ (for one suppose that y is an implicit function of x and for the other one change the roles).

c) Say which the integrals above are improper and why. Finally evaluate both.

6. Consider the region $\mathfrak{R} = \{(x,y) \mid x \geq 1, 0 \leq y \leq \frac{1}{x}\}$. If $\mathfrak{R}$ is rotated about the x-axis, calculate the volume of the resulting solid and show that the surface are is infinite.

7. Find the coordinates of the centroid of the region bounded by $y = \sin(x)$, $y = \cos(2x)$ $x = \pi/6$ and $x = 5\pi/6$.